Matjaz ZUNKO, B.Sc.
University of Maribor
Faculty of Economics and Business Maribor, Slovenia
E-mail: matjaz.zunko@uni-mb.si
Timotej JAGRIC, PhD
University of Maribor
Faculty of Economics and Business Maribor, Slovenia
E-mail: timotej.jagric@uni-mb.si

TESTING FOR A STRUCTURAL BREAK IN DYNAMIC CONDITIONAL CORRELATION MODELS

Abstract. *In this paper we propose an approach for testing a structural break in dynamic conditional correlation models that is based on the extension of ordinary model equation with differential parameters, combined with dummy variables. This enables us to directly observe the difference in individual parameter or in a combination of parameters before and after the break and offers us a lot of testing opportunities. We make an extensive Monte Carlo simulation experiment of the proposed methodology for both the DCC and the ADCC multivariate GARCH models which indicate good small sample properties. In the empirical part we perform the test on the daily data of some European stock indices where we find a significant structural break in both the long-run mean and the dynamic part of the models after the introduction of the currency euro.*

Keywords: *structural break, dynamic conditional correlation, multivariate GARCH, dummy variable, Monte Carlo simulation.*

JEL classification: C32, C53, G15

1. Introduction

Common assumption of applied time series analysis and forecasting is the stability of the parameters of a model. But if a structural break occurs during the sample period, ignoring a change in parameters can lead to unsuitable model and can result in incorrect statistical inference.Hansen (2001) state that ignoring structural breaks, »inferences about economic relationships can go astray, forecasts can be inaccurate, and policy recommendations can be misleading or worse.«

As documented by Andreou and Ghysels (2008), there is a strong evidence for the occurrence of a structural break in financial time series. They state several empirical studies that report there are structural breaks in financial markets which affect returns and volatility,the shape of the option implied volatility smile, asset allocation, the equity premium, the tail of the distribution and risk management measures, as well as credit risk models and default measures.

Modeling time-varying volatilities and correlations in financial time series has grown to an extended field of applied econometrics, withnumerous variants of proposed modeling structure (Silvennoinen and Teräsvirta 2008). In the area of the multivariate GARCH models, the most commonly usedis thedynamic conditional correlations (DCC) model of Engle (2002). Its popularity arises from the simple formulation of the quasi-correlation model that is similar to the univariate GARCH equation, itspossibility of the application on high-dimensional data sets, and its simplicity of imposing the positive definiteness. It accounts for heteroskedasticity problems directly as it estimates the correlation coefficients of standardized residuals. Cappiello et al. (2006) proposed a generalization of the DCC model,which is, similar to the asymmetric univariate GARCH models, capable of modelingthe asymmetry in conditional correlations – the asymmetric dynamic conditional correlations (ADCC) model.

An occurrence of a structural break in financial time series is a topic that appears in every field of econometric modeling. Univariate conditional volatility models are well studied on this topic (e.g. Hillebrand 2005)but there is less literature about observationand analysis of a structural break in time-varying correlations. It seems as they started to appear only recently. There are only few studies that include a structural break in the DCC or ADCC model (e.g. Kearney and Potì2006, Hyde et al. 2007, Li 2008) and most of them follow the procedure proposed by Cappiello et al. (2006).In their article, they have beside the proposal of the asymmetric approach to dynamic correlations and the generalized version of the model, proposed also the procedure to test a structural break in the model. They use a dummy variable in the dynamic conditional correlation model equation for the distinction of intercept terms or for the model as a whole.

With this approach,a researcher is only limited to a likelihood ratio testing procedure for exploring the significance of a structural break. In this paper we propose an extension of dynamic conditional correlation model equations with differential parameters, combined with dummy variables. These differential parameters indicate directly by how much the parameters in period after the structural break differ from the parameters before the structural break. They allow us to perform *t*-tests about the change of individual parameter and Wald tests about the change of a combination of parameters. Instead of breaking the whole dynamic part of a model, we can let a structural break in only several dynamic parameters and separately statistically infer about such change. The proposed approach can have an effect on both the empirical results and on their subsequent interpretation. Particulary interesting is a changein thepersistence of the DCC model and we are able to interpret the change in only the asymmetric parameter of the ADCC model after the structural break.

We make an extensive Monte Carlo simulation investigation of the proposed methodology. We try to answer the question what consequences a structural break in the volatility of univariate series has on the estimation of the correlations and if we should account it in the estimation procedure. The results show robustness of the de-garching part of the proposed methodology at repeating simulations. Before performing a statistical inference, we should check for normality of the de-garched series since for normally distributed innovations, the covariance estimation with usual maximum likelihood's outer product of gradientsis preferable over quasi-maximum likelihood's sandwich estimation. For nonnormal innovations we make the latter procedure of statistical inference but should supplement conclusions with the first one.

The empirical part of the paperupgrades results of several other papers that investigate the impact of the introduction of the euro on international equity markets. Besides confirming the discovery that the structural break in the long-termmeanis present, we as well observed the structural break in the dynamics.European equity markets became more correlated and even resistant to joint bad news. The restricted models with constant parameters include spurious persistence before the euro.

The rest of the paper is structured as follows. First we presents the idea of adding dummy variables into dynamic conditional correlation model equations, testing procedure is described and other methodological issues are discussed. The next section reports extensive Monte Carlo simulations for the levels and the powers of the proposed testing procedure. In section 4 we apply the presented methodology on empirical data. Section 5 discusses and concludes.

2. Methodology

To test whether a structural break has a significant impact on the model of dynamic conditional correlations we follow the DCC methodology of Engle (2002) and it's generalization to the asymmetricADCC methodology of Cappiello etal. (2006). These models assume that returns from k assets are conditionally multivariate normal with zero expected value and covariance matrix $\mathrm{H_t}$ (Cappiello etal. 2006):

$$r_t|\Im_{t-1}\sim N(0,H_t), \tag{1}$$

where $\Im_{t-1}$ is the information set available at time $t-1$. They use the fact that H_t can be decomposed into variance and correlation parts as (Cappiello etal. 2006):

$$H_t = D_t R_t D_t, \tag{2}$$

whereD_t is the diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{it}}$ on the i th diagonal, and R_t is the time varying correlation matrix. The estimation is done in two stages. In the first stage, we first fitthe univariate volatility models to each of the assets and estimate their standard deviations $\sqrt{h_{it}}$,that are then used to construct standardized residuals or volatility-adjusted returns $\varepsilon_{it} = r_{it}/\sqrt{h_{it}}$(Engle 2009). Cappiello etal. (2006) stated that for this de-garching process any univariate GARCH process that is covariance stationary and assumes normally distributed errors can be used. In the later stages of model estimation we assume that the volatility models are correctly specified. If this assumption does not hold, the correlation estimates will notbe consistent (Cappiello etal. 2006). To minimize the risk that the univariate models will lead to inconsistent correlation estimates they estimated several GARCH models and then selected the best one using the Bayesian information criterion(BIC). We used a similar approach by applying the following models on every asset (all with one lag of volatility, one lag of innovation and if the model includes it, one lag of negative innovation):

1. AGARCH (Engle 1990),
2. AVGARCH (Taylor 1986),
3. EGARCH (Nelson 1991),
4. GARCH (Bollerslev 1986),
5. GJR-GARCH (Glosten et al. 1993),
6. NAGARCH (Engle and Ng 1993),
7. TGARCH (Zakoian 1994),
8. VGARCH (Engle and Ng 1993).

In the second stage, the parameters of the dynamic conditional correlations are estimated.Engle (2002) and Cappiello etal. (2006) used the correlation targeting approach to estimate the intercept parameters of the conditional correlation process. Engle (2009) stated that this procedure is only an approximation but that the estimator is consistent and that it substantially reduces the number of remaining unknown parameters. For correlation targeting, the unconditional covariance $\bar{R}$of the volatility-adjusted returns ε_{it} and the negative part of the unconditional covariance $\bar{N}$ of the specificvolatility-adjusted returns $n_{it} = \min(\varepsilon_{it}, 0)$ in the ADCC model, are computed before the second stage.The structure of quasi-correlation is (Engle 2002):

$$Q_t = (1 - \alpha - \beta)\bar{R} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}, \tag{3}$$

for the DCC model and (Cappiello etal. 2006):

$$Q_t = (1 - \alpha - \beta)\bar{R} - \gamma\bar{N} + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \gamma n_{t-1}n'_{t-1} + \beta Q_{t-1}, \tag{4}$$

for the scalar ADCC model. Since quasi-correlation matrix Q_t is not necessarily the correlation matrix, we must rescale it as

$$R_t = {Q_t}^{*-1} Q_t {Q_t}^{*-1}, \tag{5}$$

where${Q_t}^*$ is a diagonal matrix with the square root of the ith diagonal element of Q_t on its ith diagonal position.

To extend the DCC and ADCC models to allow for a structural break in the parameters, we proceed with adding dummy variables as follows. Let D_t be a dummy variable that takes the value of 1 in the period after the structural break ($t \geq \tau$, where τ is the first time point after the structural break), and 0 in the period before the structural break. The breakdate is assumed to be known *a priori*and so is exogenously set at some time point of the data.The model that allows for a structural break in the interceptpart of the model and in the dynamics could be written as proposed by Cappiello etal. (2006):

$$Q_t = [(1-\alpha_1-\beta_1)\bar{R}_1-\gamma_1\bar{N}_1](1-D_t)+ [(1-\alpha_2-\beta_2)\bar{R}_2-\gamma_2\bar{N}_2]D_t + [\alpha_1\varepsilon_{t-1}\varepsilon'_{t-1}+\gamma_1 n_{t-1}n'_{t-1}+\beta_1 Q_{t-1}](1-D_t)+[\alpha_2\varepsilon_{t-1}\varepsilon'_{t-1}+\gamma_2 n_{t-1}n'_{t-1}+\beta_2 Q_{t-1}]D_t, \quad (6)$$

wherethe separate estimates of both the full covariance matrix of volatility-adjusted returns and the negative part of it are needed before and after the break. But this model simply divides the structure of quasi-correlation into two parts and as such does not offer us much of testing opportunities.

We extend the ADCC model in a way that the parameters of the second period α_2, β_2 and γ_2 are sums of the parameters from the first period and thedifferentialparameters:

$$\alpha_2 = \alpha_1+\alpha_D, \beta_2 = \beta_1+\beta_D, \gamma_2 = \gamma_1+\gamma_D. \quad (7)$$

These differential parameters indicate to us by how much the parameters in period after the structural break differs from the parameters before the structural break. They allow us to test directly if the differential parameters are statistically different from zero when we are interested in testing the parameter change from one period to another. If we insert these forms into upper equation (6) and rearrange, we get

$$Q_t = [(1-\alpha_1-\beta_1)\bar{R}_1-\gamma_1\bar{N}_1](1-D_t)+[(1-(\alpha_1+\alpha_D)-(\beta_1+\beta_D))\bar{R}_2-(\gamma_1+\gamma_D)\bar{N}_2]D_t+(\alpha_1+D_t\alpha_D)\varepsilon_{t-1}\varepsilon'_{t-1}+(\gamma_1+D_t\gamma_D)n_{t-1}n'_{t-1}+(\beta_1+D_tB_D)Q_{t-1}, \quad (8)$$

from where it is evident that depending on the period before or after the structural break we calculate without or with differential parameters in the model. On the other side, we are not always interested in a structural break in all of the parameters but maybe only in some of them. For these purposes we can write a general form of our model with the dummy variablesspecific to every parameter that takes part in the quasi-correlation structure:

$$Q_t = [(1-\alpha_1-\beta_1)\bar{R}_1-\gamma_1\bar{N}_1]\left(1-D_{intr,t}\right)+\left[\left(1-(\alpha_1+D_{\alpha,t}\alpha_D)-(\beta_1+D_{\beta,t}\beta_D)\right)\bar{R}_2-\left(\gamma_1+D_{\gamma,t}\gamma_D\right)\bar{N}_2\right]D_{intr,t}+(\alpha_1+D_{\alpha,t}\alpha_D)\varepsilon_{t-1}\varepsilon'_{t-1}+\left(\gamma_1+D_{\gamma,t}\gamma_D\right)n_{t-1}n'_{t-1}+\left(\beta_1+D_{\beta,t}\beta_D\right)Q_{t-1}, \quad (9)$$

where $D_{intr,t}$ is 1 for $t \geq \tau$ at the assumption that the intercept part can change and 0 otherwise, $D_{\alpha,t}$ is 1 for $t \geq \tau$ at the assumption that parameter α can change and 0 otherwise, $D_{\beta,t}$ is 1 for $t \geq \tau$ at the assumption that parameter β can change and 0 otherwise and $D_{\gamma,t}$ is 1 for $t \geq \tau$ at the assumption that parameter γ can change and 0 otherwise. With these parameter specific dummy variables we have a necessary condition that to allow for a structural break in any of the parameters α, β or γ, we must allow for a structural break in the intercepts, since as soon as we have any of these dynamic parameters different in two periods, we also have different correlation targeting estimates in two periods.An advantage of this model is that although we assume that a structural break occurredduring the period of interest, we still use one model on the whole data and thus are likely to have high relative precision of the estimated parameters. Very similar model, except that we don't have the part of the quasi-correlation structure that relates to the asymmetric effect of the negative returns is for the DCC model.

Cappiello et al. (2006) stated that necessary and sufficient condition for Q_t to be positive definite in the scalar ADCC model, beside positive values of dynamic parameters α, β and γ, is that $\alpha + \beta + \delta\gamma < 1$, where δ is maximum eigenvalue of the matrix $\bar{R}^{-1/2}\bar{N}\bar{R}^{-1/2}$.Such condition in the DCC model is $\alpha + \beta < 1$. When we are estimating our model that allow for a structural break, we must be careful that these conditions hold in both periods, that is for the basic parameters and for the parameters with differential parts added.

Engle and Sheppard (2001) establishedproofs for consistency and asymptotic normality of the DCC parameter estimates, based on the proof of Newey and McFadden (1994) for two-stage GMM estimators. We can use the same arguments and thequasi-maximum likelihood (QML) »sandwich« covariance estimators since the differential parameters in our model do not affect the set of regularity conditions that are established for general group of two-stage parameter estimates $\hat{\theta}_n = (\hat{\phi}_n, \hat{\psi}_n)$ (Engle and Sheppard, 2001):

$$\sqrt{n}\left(\hat{\theta}_n - \hat{\theta}_0\right) \sim_A N\left(0, {A_0}^{-1} B_0 {A_0}'^{-1}\right), \qquad (10)$$

whereA_0 is the block matrix of Hessian matrices

$$A_0 = \begin{bmatrix} \nabla_{\phi\phi} ln f_1(\phi_0) & 0 \\ \nabla_{\phi\psi} ln f_2(\phi_0) & \nabla_{\psi\psi} ln f_2(\phi_0) \end{bmatrix} \qquad (11)$$

andB_0 is the outer product of gradients (OPG) estimator

$$B_0 = var\left[\sum_{t=1}^{n}\left(n^{-\frac{1}{2}}\nabla'_{\phi} ln f_1(r_t, \phi_0), n^{-\frac{1}{2}}\nabla'_{\psi} ln f_2(r_t, \phi_0, \psi_0)\right)\right]. \qquad (12)$$

With the appropriate choice of assumptions for which dynamic parameters can change, we have a class of nested models included in the general formulation of the quasi-correlation structure (9), from an ordinary ADCC model to the ADCC model that allows for a structural break in the intercept part and all of the dynamic parameters. Because of this nesting property, we can use different testing procedures for a variety of hypotheses about a structural break.Our structure for quasi-correlations is specially convenient since it enables us to not only answer the question of whether the model as a whole has changed after the structural break but can also pinpoint the source of the difference. Namely with only one estimation of the unrestricted model, we have parameter estimates and an estimate of their asymptotic covariance so we are able to test the hypothesis of the stability for a dynamic part of a model as a whole

$$H_0: \alpha_D = \beta_D = \gamma_D = 0 \tag{13}$$

with a Wald statistic

$$W = (\alpha_D, \beta_D, \gamma_D) Est.Asy.Var[(\alpha_D, \beta_D, \gamma_D)](\alpha_D, \beta_D, \gamma_D)', \tag{14}$$

that has a chi-squared distribution with 3 degrees of freedom. Beside this, we can look at the *t*-test statistics on individual difference parameters to gain knowledge about explicitly which of the coefficients have changed. Following the procedure of a Wald test, also other sets of hypotheses for a function of parameters $H_0: c(\psi_0) = q$ can be done. We can take similar steps for the DCC model.

When infering aboutthe zero hypothesis thatinclude an assumption about the intercept terms, we are not able to use a Wald test or a*t*-test since theasymptoticcovariancesof the intercept estimates are not directly observable. For these purposes we must estimate the restricted and the unrestricted model and then take a likelihood ratio testing procedure. The likelihood ratio test statistic has a large sample distribution equal to chi-square with degrees of freedom equal to the number of restrictions imposed. Engle and Sheppard (2001) stated that in a two-stage estimation procedure,as used in our model, the likelihood ratio test statistic will as asymptotic distribution rather than χ_r^2 have a weighted sum of r independent χ_1^2 variables.The weights are based on the limiting distribution of the parameters (Engle and Sheppard 2001).

If we assume that a structural break has a significant impact on the dynamic correlation parameters, the question arises if it has also a significant impact on the volatility processes and what consequences has this on the correlation parameter estimates in the second stage. Should we also allow to the de-garching model parameters to be able to change after a structural break? Namely the parameters of the de-garching process could have changed after the structural break and so the de-garched series would be different, leading to different estimates for the dynamics. In both cases, if we allow such change of volatility processes or not,methodology iscompatible with the theory that the de-garched series in the second stage of the estimation have conditional volatilities equal to 1. So that we would not be restricted only to the correlation parameters change, we include a similar dummy variableD_t in de-garching models and take an extensive simulation experiments to answer this question. A detailed explanation of the simulation experiments is in the next chapter.

Inclusion of a dummy variable into the de-garching processes allows a researcher to make similar inferences about a change in the volatility parameters as for the correlation parameters since every parameter in the second period is a sum of a parameter from the first period and a difference parameter. But since we are interested in the correlations in this paper, we do not report any of the results about a change in the volatility model parameters.

3. Monte Carlo simulation experiments

In this section we conduct simulation experiments for a bivariate case and discuss some obstacles that we encounter.The purpose ofthe first part of experiments is focused on the question if allowing a change in dynamic volatility processes (in a similar way as in the dynamic correlation process) has any impact on the results of a structural break in the correlations. Although literature about a structural break in the GARCH models deals mainly about a significant change in the constant parameter ω, we simulate and in the estimation allow all of the GARCH parameters to change. For instance the GARCH(1,1) model equation is extended as

$$h_t = (\omega_1 + D_t\omega_D) + (\alpha_1 + D_t\alpha_D)\varepsilon_{t-1}{}^2 + (\beta_1 + D_t\beta_D)h_{t-1} \tag{15}$$

and other univariate de-garching model equations are extended similarly.For such extension, we must be careful that models suffice parameter constraints for stationarityin both periods, before and after the break (several of these conditions can be found in Carrasco and Chen 2002).

We made 1000 simulations and we use the sample size $T = 1000$, which corresponds to 4 years of daily data and is empirically at most useful. We used the following GARCH(1,1) DPG for volatility processes:

$$h_{1,t} = \left[5 \cdot 10^{-6} + 0.05r_{1,t-1}^2 + 0.94h_{1,t-1}\right](1 - D_t) + \left[3 \cdot 10^{-5} + 0.25r_{1,t-1}^2 + 0.74h_{1,t-1}\right]D_t, \tag{16}$$

$$h_{2,t} = \left[5 \cdot 10^{-6} + 0.15r_{1,t-1}^2 + 0.75h_{1,t-1}\right](1 - D_t) + \left[1 \cdot 10^{-6} + 0.05r_{1,t-1}^2 + 0.85h_{1,t-1}\right]D_{t,} \tag{17}$$

$$r_{1,t} = \sqrt{h_{1,t}}\varepsilon_{1,t},\ r_{2,t} = \sqrt{h_{2,t}}\varepsilon_{2,t}, \tag{18}$$

where in the first experiment the innovations are drawn from the normal distribution

$$\begin{pmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{pmatrix} \sim N\left[0, \begin{pmatrix}1 & \rho_t\\ \rho_t & 1\end{pmatrix}\right], \tag{19}$$

and in the second experiment the innovations are drawn from thestandardized Student's*t*-distribution with eight degrees of freedom

$$\begin{pmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{pmatrix} \sim t_8\left[0, \begin{pmatrix}1 & \rho_t\\ \rho_t & 1\end{pmatrix}\right]. \tag{20}$$

We set the breakdate at $\tau = 501$ to devide the sample period exactly into halves and the dummy variable $D_t$has similar definition as in equation (9). If the processes (16) and (17) change, the first oneproducesmore volatile while the second oneproducesless volatile series after the structural break.The values of returns with such DGPs correspond tothe values of empirical daily data series.The motivation to simulate innovations from both the normal distribution and the Student's *t*-distribution is a simulation experiment of multivariate volatility models from Hafner and Herwartz (2008) who indicated that different innovation's distribution assumption prefers different covariance estimation procedure.Similar as in their study, we check if there are any differences in the levels if we compute asymptotic covariances of parameter estimates with the quasi-maximum likelihood's (QML) sandwichprocedure ($\bar{J}^{-1}\bar{I}\bar{J}^{-1}$) or with direct, more conventional maximum likelihood's outer product of gradients (OPG) procedure($\bar{I}^{-1}$).

The quasi-correlations were simulated for the whole period with following DGP for the ADCC experiments

$$Q_t = \left[(1-0.05-0.89)\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix} - 0.05\begin{bmatrix}0.5 & 0.3\\ 0.3 & 0.5\end{bmatrix}\right] + 0.05\varepsilon_{t-1}\varepsilon'_{t-1} + 0.05n_{t-1}n'_{t-1} + 0.89Q_{t-1}, \quad (21)$$

and with this DGP for the DCC experiments

$$Q_t = \left[(1-0.06-0.9)\begin{bmatrix}1 & 0.5\\ 0.5 & 1\end{bmatrix}\right] + 0.06\varepsilon_{t-1}\varepsilon'_{t-1} + 0.9Q_{t-1}. \quad (22)$$

All together there are 16 experiments, 8 for the ADCC model and 8 for the DCC model: for every innovation's distribution assumption we conducted4 types of estimations. In the first (I) we simulated a structural break in the volatility processes and allowed the parameters of the de-garchingmodels to change. In the second type of estimations (II) we again simulated a change in the volatilities and then estimatedthe de-garchingmodels with constant parameters for the whole period. The third and the fourth estimations are for DGPs without volatility processes change and with(III) or without (IV) allowance to the de-garchingmodels parameters to be able to change.

In the estimation we followedthe procedure described in the methodological section of the paper. At de-garching univariate series, we choosed with the Bayesian information criterion only from those processes that do not have any active constraint on the parameters. Namely, GARCH models with active constraints do not necessarilyhave positive definite Hessian matrix at MLE parameter estimates and consequently the estimated covariance of the whole model, which includes this block in the sandwich calculation procedure, is not necessarily positive definite. Also the correction of the likelihood ratio test statistics could have problems with negative eigenvalues because of this poor definiteness. Within the de-garching processes at least constrained are the EGARCH and the VGARCH model with only weak condition of $|\beta| < 1$ and in all simulationsit turned out that at least one of these models is without active constraints. Despite this restriction we still observed some negative eigenvalues at the likelihood ratio test statistic corrections (around 17% of simulations at the ADCC model and 13% at the DCC model) that have probably source in numerical errors at matrix inversion (some of the matrices are close to singular). Similar simulation anomalies are reported in the paper of Carnero et al. (2004). In such situations we computed the P-value of the statistic directly from χ_r^2 distribution.

In simulation experiments we observed some replicates at which dynamic conditional correlation model parameters are estimated to be on the lower bound that is set in the calculation (the requirement of the model is that they are positive which is achieved with setting a small positive number as their lower bound). In such cases we could face with similar problems as at univariate models that the Hessian at such parameter estimates is not positive definite. So we proceeded in a way that we set this parameter as insignificantly different from zero (for inference purposes) and then calculated Hessian and scores at other, reduced in size, parameters.

To avoid initialization effects, the first 200 observations were discarded. We can observe that the null hypotheses of no difference in the parameters after a structural break in our experiments corresponds to the true DGP and so the empirical rejection frequencies should be near the nominal level of significance. We performed all of the experiments at the conventional 0.05 level of significance. We only reported the levels for the most interesting statistics: the Wald test for the stability of the dynamic part of the model (W), the *t*-test on individual differential parameters, the likelihood ratio test for the stability of the whole model (LRT M), the likelihood ratio test for the stability of the intercept (LRT Intr) and the likelihood ratio test for the stability of individual parameters. All numerical calculations were carried out on Matlab (R2011a) with program codes which we developed.

Table1. The levels of the ADCC model simulation, normal innovations

	I S	II S	III S	IV S	I OPG	II OPG	III OPG	IV OPG
W	0.16	0.16	0.16	0.16	0.05	0.04	0.04	0.04
t-test α_D	0.08	0.09	0.07	0.08	0.05	0.06	0.04	0.06
t-test β_D	0.09	0.08	0.09	0.08	0.05	0.04	0.04	0.04
t-test γ_D	0.07	0.06	0.07	0.07	0.04	0.04	0.04	0.04
LRT M	0.08	0.06	0.09	0.06				
LRT Intr	0.10	0.06	0.09	0.07				
LRT α_D	0.10	0.07	0.10	0.09				
LRT β_D	0.10	0.07	0.10	0.09				
LRT γ_D	0.08	0.06	0.08	0.07				

Note: The numbers represent the actual rejection frequencies in 1000 replications based on the nominal 5% level of significance. S stands for QML sandwich covariance estimates and OPG stands for maximum likelihood OPG covariance estimates. Numbers I – IV label different combinations for thesimulation and the estimation of volatility parameters prepositions: at I and II we simulated a change in volatility process and in III and IV we simulated series with constant parameters; at I and III we then estimated de-garching models with an allowance of parameters to change and in II and IV we estimated models with constant parameters for the whole period. W stands for the Wald test for the stability of the dynamic part of the model, the *t*-tests are taken on individual differential parameters and LRTs standasthe likelihood ratio tests for the stability of the whole model (LRT M), the likelihood ratio tests for the stability of the intercept (LRT Intr) and the likelihood ratio tests for the stability of individual parameters (LRT α_D, LRT β_D and LRT γ_D).

The results indicated that the structural break in the volatilities does not have much impact on the levels of the statistics about thecorrelation model parameters. The levels of the statistics at the experiments where we de-garched univariate series with an ordinary GARCH models exceededthe nominal level of significance by a little lower range,regardless of the presence of a simulated change in univariate DGP or not.

Table 2. The levels of the ADCC model simulation, Student'st_8 innovations

	I S	II S	III S	IV S	I OPG	II OPG	III OPG	IV OPG
Wald	0.18	0.17	0.18	0.17	0.11	0.10	0.11	0.13
t-test α_D	0.08	0.09	0.08	0.08	0.08	0.07	0.08	0.08
t-test β_D	0.10	0.11	0.10	0.09	0.08	0.09	0.10	0.10
t-test γ_D	0.07	0.08	0.07	0.07	0.09	0.09	0.08	0.10
LRT M	0.16	0.12	0.16	0.12				
LRT Intr	0.14	0.10	0.14	0.09				
LRT α_D	0.11	0.10	0.13	0.11				
LRT β_D	0.13	0.11	0.14	0.12				
LRT γ_D	0.10	0.08	0.09	0.09				

Note: For an explanation of the labels and meaning of the results see the note under Table 1.

Table 3. The levels of the DCC model simulation, normal innovations

	I S	II S	III S	IV S	I OPG	II OPG	III OPG	IV OPG
Wald	0.13	0.12	0.14	0.12	0.02	0.03	0.02	0.03
t-test α_D	0.08	0.09	0.08	0.09	0.04	0.04	0.03	0.04
t-test β_D	0.10	0.10	0.11	0.10	0.03	0.03	0.03	0.03
LRT M	0.09	0.05	0.08	0.05				
LRT Intr	0.11	0.06	0.11	0.07				
LRT α_D	0.09	0.08	0.08	0.07				
LRT β_D	0.09	0.07	0.10	0.08				

Note: For an explanation of the labels and meaning of the results see the note under Table 1.

Table 4. The levels of the DCC model simulation, Student'st_8 innovations

	I S	II S	III S	IV S	I OPG	II OPG	III OPG	IV OPG
Wald	0.14	0.13	0.13	0.13	0.07	0.07	0.07	0.07
t-test α_D	0.10	0.10	0.09	0.09	0.08	0.09	0.08	0.09
t-test β_D	0.13	0.12	0.12	0.11	0.07	0.07	0.07	0.06
LRT M	0.14	0.10	0.14	0.10				
LRT Intr	0.12	0.09	0.13	0.08				
LRT α_D	0.11	0.11	0.11	0.12				
LRT β_D	0.15	0.14	0.14	0.14				

Note: For an explanation of the labels and meaning of the results see the note under Table 1.

As the second criterion for evaluation of the de-garching procedure we compared biases for the correlation parameter estimates across experiments. In the ADCC model simulations we found that the coefficientsα and β were slightly biased downward and that the bias was a tiny lower if we degarched with an ordinary GARCH models. The parameter γ was biased upward while the difference in the bias between different de-garching models depend on the presence of the structural break in simulated volatility processes but was small again. In the DCC model simulations the estimations ofthe coeffient α were centered around the simulated value without any difference in the de-garchingprocesses and the estimates of the coefficient β are slightly biased downward with a little lower bias at the ordinary de-garcing process.

The third evaluation criterion is mean absolute error between the simulated dynamic conditional correlation series and the estimated one:

$$MAE = \frac{1}{T}|\sum \rho_t - \hat{\rho}_t|. \tag{23}$$

In all simulation experiments it turned out that the mean absolute error was lower when we de-garched univariate series with an ordinary GARCH models. MAE was lower at ordinary de-garching in around 52% of replications if we simulated the structural break in the volatility process and in around 58% of replications if we simulatedwith stable GARCH parameters.

All three criteria favor an ordinary GARCH models in the de-garching part of the model estimation. It is better if we do not complicate these models and use their standard equations. The reason for such small differences could be too small change of the volatility process that does not have enough impact, although we think that the one we simulated is a quite large. Another argument to treat the volatility model parameters as constant although we assume a structural break in the conditional correlations is that if we estimate the extended GARCH models it happens more often that there is an active constraint for the estimated model parameters and so we have less models to choose from with the BIC. Such small differences between the experiments in a way indicated a robustness of the dynamic conditional correlation models to the de-garching part of the estimation at repeating simulations.

The levels in tables 1 – 4 mainly exceededthe nominal level 0.05, especially levels of the Wald tests and likelihood ratio tests. These exceedances are analogue with the exceedances reported by Nakatani and Teräsvirta (2009) in their Monte Carlo simulations of the test for volatility interactions. When they simulated series with the constant conditional correlations model of Bollerslev (1990) they encountered levels close to nominal. But when they tried to simulate series with changing conditional correlations they encountered higher levels, similar to ours. The size of their level exceedances did not decrease with increasing sample sizes. We think that the reason for this is that the series are distorted twice – with the correlation process and with the volatility process. Information about the original parameter values is therefore more blurred, which affects specially the likelihood ratio tests where values of the log-likelihood function are concealed by the lack of identification of certain parameters under the restricted and unrestricted model (Hansen 1991). With this argument we assessed the results of the simulations as indication of good small sample properties of our testing procedure for a structural break in dynamic conditional correlation models. When we checked other conventional levels of significance,the 1% and the 10% levels, the difference in the levels between experiments were similar and led to the same conclusions.

Comparison of the QML sandwich and the OPG inferences indicated overrejection of the null hypotheses at the QML and some over- and underrejection of the null hypotheses at the OPG covariance estimates. Slight underrejection was present at the normal innovations distribution assumption where the difference between estimation procedures was larger, more at the Wald test statistics as at the *t*-statistics. The source of different levels could be observed if we looked at the histograms of these P-values. The QML based were balanced toward zero with observable outstanding column at values smaller than 1% while the OPG based were more uniformly distributed which corresponds to their random simulation source. QML sandwich covarianceswere underestimated for these simulation experiments.

These results suggest a use of the OPG procedure for normal innovations. The inaccuracy of the QML calculations at conditional normality was more evident at the correction of the likelihood ratio test statistics. If we corrected the distribution of these tests with the procedure proposed by Engle and Sheppard (2001) then the levels exceed the nominal significance even more (1 to 3 percentage points) as if we calculated P-values from the uncorrected distributions. The levels in tables 1 and 3 are therefore reported for statistics without the correction of distribution. Based on these observations for the assumption of conditional normality we can confirm the statement of Hafner and Herwartz (2008) that the QML based P-values should be preferred over the OPG counterparts only if diagnostic tests indicate nonnormality of the de-garched data that enters the correlation part of estimation. This diagnostic checking (e.g. Jarque-Bera test) should therefore be a part of the estimation procedure.

Majority of empirical studies report a nonnormal distribution of returns, therefore the simulation results at Student's *t*-distribution of innovationshave more practical value. For this assumption the levels were also closer to the nominal significance at the OPG based as at the QML based P-values but the difference was smaller. The levels of the corrected likelihood ratio test statistics were closer to the nominal at ADCC simulations and more distant at DCC simulations. The reason for such vague results for nonnormal distribution assumption could be in the use of numerical derivatives in the calculation. Hafner and Herwartz (2008) advised not to use them for an empirical practice at the QML inference where we have to calculate both the scores and the Hessians. On the other side they reported there was almost no difference between the numerical and the analytical approach at the OPG covariance based inferences.

In repeating simulation calculations these differences result in different levels that favor the OPG inference. At individual calculation we think one should withstand by the theory and use the QML sandwich inference but should perform the OPG inference as well and then check if the conclusions are similar or not. Supplement use of the OPG inference should be used especially at weak QML sandwich inferential results with P-values near the level of significance. Our empirical experiences support this use of both inference procedures.

In the second part of the simulation experiments we checked the powers of the proposed tests. Based on the results of the levels we only simulated volatility series without a structural break, we de-garched with the standard models and we only took innovations from the standardized Student's *t*-distribution with 8 degrees of freedom.DGPs were similar as described above; the relevant ADCC and DCC parameters for each experiment are presented in the first row of the tables below. We again made 1000 simulations with the sample size $T = 1000$ and the break date is at the half of simulated series.

We simulated several DGPs where only the parameters of interest change. But since this changes the persistence in models, we simulated the models with maintained level of persistence as well. Becausethe powers depend on the size of the effect among other factors, we simulated DGPs with quite a big change in the parameters but still observable in empirical studies. In tables 5 and 6 we report the powers of relevant test statistics: the Wald test for the stability of the dynamic part of the model (W), the *t*-test on individual differential parameters and the likelihood ratio tests for the stability of the whole model (LRT M) and for the stability of individual parameters.The level of significance is 5%.

Table 5. The powers of the ADCC model simulation, Student's t_8 innovations

	DGP changes in α	DGP changes in β	DGP changes in γ	DGP changes in α and β
	$\alpha_1 = 0.15$ $a_2 = 0.04$ $\beta = 0.8$ $\gamma = 0.04$	$\alpha = 0.04$ $\beta_1 = 0.9$ $\beta_2 = 0.6$ $\gamma = 0.04$	$\alpha = 0.04$ $\beta = 0.8$ $\gamma_1 = 0.15$ $\gamma_2 = 0.04$	$\alpha_1 = 0.15$ $a_2 = 0.04$ $\beta_1 = 0.8$ $\beta_2 = 0.91$ $\gamma = 0.04$
W	0.76	0.32	0.29	0.75
t-test α_D	0.46			0.58
t-test β_D		0.25		0.40
t-test γ_D			0.12	
LRT M	0.94	0.30	0.29	0.79
LRT α_D	0.89			0.75
LRT β_D		0.41		0.52
LRT γ_D			0.31	

Note: The numbers represent actual rejection frequencies in 1000 replications based on the nominal 5% level of significance. Covariance estimates are based onthe QML sandwich procedure. W stands for the Wald test for the stability of the dynamic part of the model, *t*-

tests are taken on individual differential parameters and LRTs stand as the likelihood ratio tests for the stability of the whole model (LRT M) and the likelihood ratio tests for the stability of individual parameters (LRT α_D, LRT β_D and LRT γ_D).

The results showed reasonable powers for a change in α and more modest powers for a change in β.The rejection frequencies for a change in γ indicated small powers so such tests are prone to be inconclusive. Wald tests were generally quite powerful for a detection of a change. The powers of likelihood ratio tests were higher as corresponding *t*-tests.

Table 6. The powers of the DCC model simulation, Student's t_8 innovations

	DGP changes in α	DGP changes in β	DGP changes in α and β
	$\alpha_1 = 0.18$ $a_2 = 0.06$ $\beta = 0.8$	$\alpha = 0.08$ $\beta_1 = 0.9$ $\beta_2 = 0.6$	$\alpha_1 = 0.18$ $a_2 = 0.04$ $\beta_1 = 0.8$ $\beta_2 = 0.94$
W	0.90	0.45	0.96
t-test α_D	0.64		0.91
t-test β_D		0.39	0.70
LRT M	1.00	0.82	0.98
LRT α_D	0.97		0.95
LRT β_D		0.80	0.79

Note: For an explanation of the labels and meaning of the results see note under Table 5.

4. Empirical performance

We applied tests for a structural break in the ADCC model to the equity index returns data of three large European countries: France (CAC40), Germany (DAX30) and United Kingdom (FTSE100). These countries have similar trading hours so we had to synchronize data only for market closure days at individual country. For these days we took the last known index value. We synchronized and analyzed data for each pair of countries so that we add less missing values and that the estimated parameters are specific for the correlations between those two countries (ADCC model can be performed on multivariate series but the model parameters are then the same for the whole panel). The sample covers the period from January 2, 1992 until December 31, 2007(approximately 4150 data points) and the breakdate is the introduction of the currency euro at the beginning of year 1999. Data series were terminated before the beginning of the current World financial crisis that brought another big change in financial markets and could produce another structural break in the models. We used the daily log prices.

With this sample we could upgrade results of several other papers that questioned the impact of fixing the exchange rates within the European Monetary Union at that time (e.g. Emiris 2004, Cappiello et al. 2006, Kearney and Potì 2006, Li 2008) who found a significant change in the long-term mean correlations. Our research is advanced since we checked if there was also a structural break in the dynamic part of the model and in individual parameters.

For all of the series we could reject the null hypothesis of a unit root with the Augmented Dickey-Fuller test and the null hypothesis of normality with the Jarque-Berra test, all with very low P-values.Best univariate GARCH specification according to BIC was the NAGARCH for CAC40, the GJR-GARCH for DAX30 and the EGARCH for FTSE100, but the differences between models were small, especially between the asymmetric dynamic volatility models. The presence of asymmetry in conditional second moments of these series were strong. After de-garching, all of the series still reported very low P-values of the Jarque-Berra test.

Table 7. ADCC model with an allowance for a structural break at every parameter at CAC40–DAX30

CAC40–DAX30	LRT M	LRT Intr	W S	W OPG
	1.52E-07	0.011	7.70E-12	1.10E-11

	α_1	β_1	γ_1	α_D	β_D	γ_D
parameter	1.00E-10	0.951	0.043	0.035	0.014	-0.043
t-test S	1	0	1.21E-03	0.093	0.373	0.041
t-test OPG	1	0	2.93E-04	5.72E-12	0.470	3.03E-03

Note: The numbers represent the estimated value of parameters or the P-values of these estimates. S stands for the QML sandwich covariance estimates and OPG stands for the maximum likelihood OPG covariance estimates. W stands for the Wald test for the stability of the dynamic part of the model, *t*-tests are taken on individual parameters, LRT M stands for the likelihood ratio tests for the stability of the whole model and LRT Intr stands for the likelihood ratio tests for the stability of the intercept.

Table 8. ADCC model with an allowance for a structural break at every parameter at CAC40–FTSE100

CAC40–FTSE100	LRT M	LRT Intr	W S	W OPG
	3.33E-16	1.29E-08	2.06E-04	1.04E-11

	α_1	β_1	γ_1	α_D	β_D	γ_D
parameter	0.002	0.930	0.064	0.048	0.014	-0.064
t-test S	0.231	0	1.28E-04	6.72E-04	0.435	3.90E-03
t-test OPG	0.394	0	1.18E-06	6.50E-08	0.406	8.58E-05

Note: For an explanation of the labels and meaning of the results see note under Table 7.

Table 9: ADCC model with allowance for a structural break at every parameter at DAX30–FTSE100

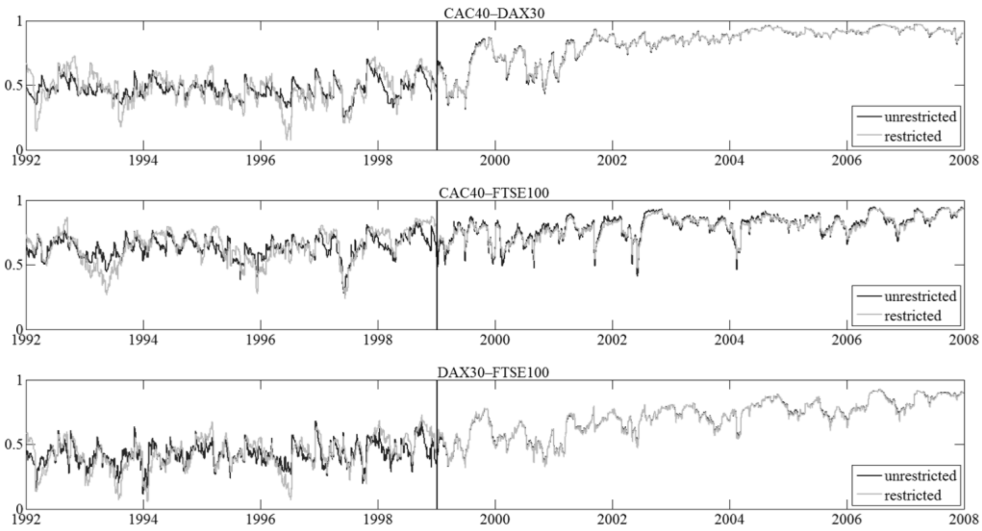
DAX30–FTSE100	LRT M	LRT Intr	W S	W OPG
	2.94E-07	3.49E-03	1.49E-03	0.102

	α_1	β_1	γ_1	α_D	β_D	γ_D
parameter	0.025	0.903	0.030	0.002	0.068	-0.028
t-test S	0.030	0	0.029	0.428	5.73E-04	0.173
t-test OPG	0.013	0	0.041	0.873	0.031	0.157

Note: For an explanation of the labels and meaning of the results see note under Table 7.

All three models hadthe structural break at testedbreakdate as was indicated by the loglikelihood ratio test for the model as a whole and for all three we could confirm a change in the intercept. Beside that we could be quite confident that the dynamic part of the models also changed since Wald tests are all highly significant. ADCC models for pairs CAC40–DAX30 and CAC40–FTSE100 had a low value for the weight of new information α in the period before a structural break, when a rise in correlationswas strongly affected by the joint bad news (high parameter γ). After the introduction of the euro, these markets became more correlated, as can be seen from Figure 1, and also less affected by the joint bad news (although in this period there was the dot-com bubble burst). The decrease in parameter γ to an infinitesimal value is significant. With the decrease of parameter γ the value of parameter αrises. This change is significant for CAC40–FTSE100 and inconclusive for CAC40–DAX30 with the QML based inference. But since the P-value is near the nominal level of significance 5%, we should check the OPG based *t*-statistic and the LRT statistic for differential parameter α_D.Both of them were highly significant so we can conclude that this parameter changed as well. The difference in parameter β is small and insignificant. The structural break in the dynamics of the ADCC model for DAX30–FTSE100 appears in different way. Parameter α tends to increase whileparameter γ tends to decrease but both differences were insignificant.The change in dynamics of this model arises from the significant increase of parameter β.

Figure 1: Dynamic correlations of the ADCC model with constant parameters and of the ADCC model with an allowance for a structural break



In Figure 1 are comparisons between the ADCC model correlations with constant parameters for the whole period and the ADCC model correlations with an allowance for a change in all of parameters. We can see that the restricted models are more influenced by the second period since series after the structural break are similar. All restricted models include spurious persistence for the first period.

5. Conclusion

When building a model on data that covers a long time period we have to be aware of the possibility for a presence of a structural break in the process. In this paper we proposed an approach for testing a presence of a structural break in dynamic conditional correlations. We extended model equations so that the parameters in the period after the break date are sums of the parameters from the base period and the differential parameters. This enables us to directly observe differences in parameters before and after the break date, we can perform *t*-tests for a significance of these differences and Wald tests for an inference about differences in combination of parameters. If statistical tests indicate a change in the model as a whole, we can pinpoint the source of the difference with only one estimation of the model. We can investigate a change in the asymmetric parameter of the ADCC model only.

We made an extensive Monte Carlo experiment of proposed model which indicated that the test statistics have favorable finite sample properties. For the de-garching part of model estimation it turned out that dynamic conditional correlation models are quite robust at repeating simulations. It is better if we do not complicate GARCH models with an allowance for a structural break in the volatilities. Distributional assumption turned out to be relevant for choosing among QML sandwich or ML OPG covariance estimation procedure and subsequent inference.

Most researchers account for a structural break in their dynamic conditional correlation models with the procedure proposed by Cappiello et al. (2006). With this methodology several papers found the significant structural break only in the long-term mean of European equity markets after the introduction of the euro. In this paper we confirmed their findings about intercepts and upgraded conclusions with statistical significant structural break in the dynamics as well. In the empirical part we also demonstrated the advantage of the proposed procedure at investigating and interpreting the difference in individual parameters.

In this paper we addressed only one break date in processes and used models with one lag only. Extensions to multiple break dates and multiple lags were left for future research. Interesting to examine would be how proposed testing procedure can help us at finding the best break date itself.

6. REFERENCES

[1]**Andreou, E. and Ghysels, E. (2008), *Structural Breaks in Financial Time Series***. In T. G. Anderson, R. A. Davis, *J.-P. Kreiss,& T. Mikosch (Eds.)*; *Handbook of financial time series* (p. 839–866*). Berlin: Springer*;

[2]**Bollerslev, T. (1986),** ***Generalized Autoregressive Conditional Heteroskedasticity***; *Journal of Econometrics*, 31, 307-327;
[3]**Carnero, M.A., Peña, D. and Ruiz, E. (2004),** ***Persistence and Kurtosis in GARCH and Stochastic Volatility Models***; *Journal of Financial Econometrics*, 2, 319–342;
[4]**Carrasco, M., Chen, X. (2002),** ***Mixing and Moment Properties of Various GARCH and Stochastic Volatility Models***. *Econometric Theory* 18, 17–39;
[5]**Cappiello, L., R. F. Engle and K. Sheppard (2006),** ***Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns***; *Journal of Financial Econometrics*, 4, 537-572;
[6]**Emiris, M. (2004),** ***Sectoral vs Country Diversification Benefits and Downside Risk***; *Working Paper, n°48*, National Bank of Belgium, May;
[7]**Engle, R.F. (1990),** ***Discussion: Stock Market Volatility and the Crash of '87***; *Review of Financial Studies*, 3, 103–106;
[8]**Engle, R. F. (2002),** ***Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models***. *Journal of Business and Economic Statistics* 20, 339–350;
[9]**Engle, R. (2009),** ***Anticipating Correlations: A New Paradigm for Risk Management***. *Princeton University Press*;
[10]**Engle, R. and Ng, V. (1993),** ***Measuring and Testing the Impact of News on Volatility***. *Journal of Finance* 48, 1749–1778;
[11]**Engle, R. F. and K. Sheppard.(2001),** ***Theoretical and Empirical Properties of Dynamic Conditional Correlation MVGARCH***.*Working Paper No. 2001–15*, University of California, San Diego;
[12]**Glosten, L., R. Jagannathan and D. Runke (1993),** ***Relationship between the Expected Value and the Volatility of the Nominal Excess Return on Stocks***; *Journal of Finance* 48, 1779–1801;
[13]**Hafner, C. M. and Herwartz, H. (2008),** ***Analytical Quasi Maximum Likelihood Inference in Multivariate Volatility Models***; *Metrika*, 67, 219–239;
[14]**Hansen, B. E. (1991),** ***Inference when a Nuisance Parameter Is Not Identified under the Null Hypothesis***; *Working Paper No. 296*, Rochester Center for Economic Research, University of Rochester;
[15]**Hansen, B. E. (2001),** ***The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity***; *The Journal of Economic Perspectives* , Vol. 15, No. 4 (Autumn), pp. 117-128;
[16]**Hillebrand, E. (2005),** ***Neglecting Parameter Changes in GARCH Models***; *Journal of Econometrics* 129, 121–138;
[17]**Hyde, S., Bredin, D., Nguyen, N. (2007),** ***Correlation Dynamics between Asia-Pacific, EU and US Stock Returns***. In: *Kim S. J., McKenzie M. (Eds.);* Asia-Pacific Financial Markets: Integration, Innovation and Challenges, International Finance Review, Vol. 8, 39-61;
[18]**Kearney, C. and Potì, V. (2006),** ***Correlation Dynamics in European Equity Markets***; *Research in International Business and Finance*, Volume 20, Issue 3, 305–321;

[19]**Li, X-M, (2008*), From the EMS to EMU: Has there Been Any Change in the Behaviour of Exchange Rate Correlation*?**,*Quantitative Economic Policy. Advances in Computational Economics*, Volume 20, Part 4, 261–273;
[20]**Nakatani, T. and Teräsvirta, T. (2009), *Testing for Volatility Interactions in the Constant Conditional Correlation GARCH Model***. *Econometrics Journal* 12, 147–163;
[21]**Nelson, D. (1991), *Conditional Heteroskedasticity in Asset Returns: A New Approach***. *Econometrica* 59, 347–370;
[22]**Newey, W. K. and McFadden, D. (1994), *Large Sample Estimation and Hypothesis Testing***; *Handbook of Econometrics*, vol. 4, *Elsevier*, North Holland;
[23]**Silvennoinen, A. and Terasvirta, T. (2008), *Multivariate GARCH Models***; *Handbook of Financial Time Series*, ed. by T. Andersen, R. Davis, J. Kreiss, and T. Mikosch. *Springer;*
[24]**Taylor, S. (1986), *Modelling Financial Time Series***. *New York: Wiley*;
[25]**Zakoian, J.-M.(1994), *Threshold Heteroskedastic Models***. *Journal of Economic Dynamics and Control* 18, 931–955.